TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 1 2022

Section 1 Calculator Free

Counting Techniques & Vector Introduction

STUDENT'S NAME _____

DATE: Thursday 24th February **TIME:** 20 minutes **MARKS**: 15

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

(a) $\dfrac{8! - 7!}{6!}$ [2]

$= \dfrac{8\times7\times6! - 7\times6!}{6!}$

$= \dfrac{6!(8\times7 - 7)}{6!}$

$= 49$

(b) Determine the integer n if ${}^{n}P_{n-7} = 720$ [3]

$\dfrac{n!}{(n-(n-7))!} = 720$ ✓

$\dfrac{n!}{7!} = 720$

$n! = 720 \times 7!$ ✓

$n! = 10\times9\times8\times7!$

$\therefore \quad n = 10$ ✓

2. (2 marks)

47 students answered a spelling quiz. One student made seven errors, with the rest making fewer errors than this student. Prove that at least seven of the students made an equal number of errors.

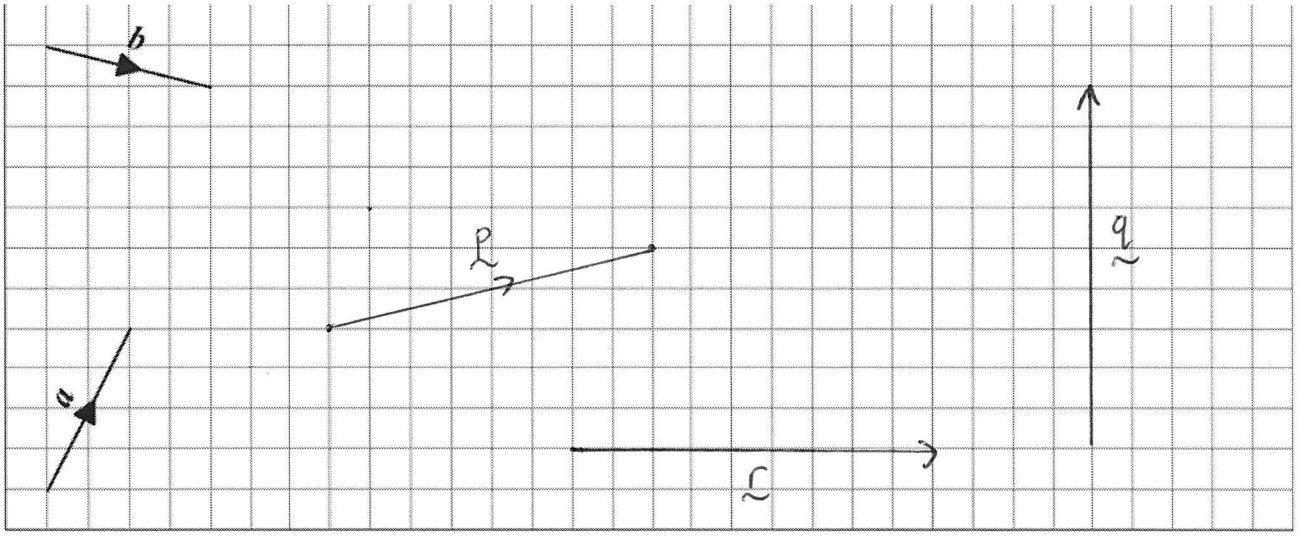
Using pigeonhole principle → 7 pigeonholes [0-6]

46 pigeons ✓

We can evenly distribute 6 students in each pigeonhole but are still left with 4 to distribute. So at least one pigeonhole has 7+ students ✓

3. (4 marks)

Vectors **a** and **b** are shown on the grid below.



On the grid above sketch the vectors **p**, **q** and **r** where:

(a) $\mathbf{p} = \mathbf{a} + 2\mathbf{b}$ ✓

(b) $\mathbf{q} = 2\mathbf{a} - \mathbf{b}$ ✓

(c) $\mathbf{r} = -2\mathbf{b} - 0.5\mathbf{a}$ ✓✓

4. (4 marks)

Prove that ${}^{n}P_{r} + r\left({}^{n}P_{r-1}\right) = {}^{n+1}P_{r}$

$$RHS = \frac{n!}{(n-r)!} + r\left[\frac{n!}{(n-(r-1))!}\right] \quad \checkmark$$

$$= \frac{n!}{(n-r)!} + \frac{rn!}{(n-r+1)!}$$

$$= \frac{n!}{(n-r)!} + \frac{rn!}{(n-r+1)(n-r)!}$$

$$= \frac{n!(n-r+1) + rn!}{(n-r+1)(n-r)!} \quad \checkmark$$

$$= \frac{n!n - n!r + n! + rn!}{(n-r+1)(n-r)!}$$

$$= \frac{n!(n+1)}{(n-r+1)(n-r)!} \quad \checkmark$$

$$= \frac{n!(n+1)}{(n-r+1)!} \quad \checkmark$$

$$= {}^{n+1}P_{r}$$

$$= LHS \qquad Q.E.D$$

TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 1 2022

Section 2 Calculator Assumed

Counting Techniques & Vector Introduction

STUDENT'S NAME _____

DATE: Thursday 24th February **TIME:** 30 minutes **MARKS**: 34

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

QUESTIONS BEGIN ON THE NEXT PAGE

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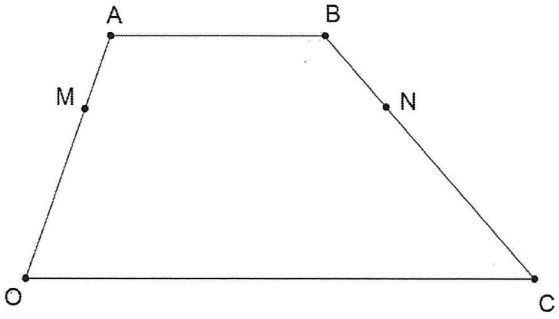
5. (4 marks)

The diagram shows trapezium $OABC$, with $\overrightarrow{OC} = 2 \times \overrightarrow{AB}$

Points M and N divide OA and CB in the ratio $OM:MA = CN:NB = 2:1$, and

Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$.



Express each of the following in terms of $\boldsymbol{a}$ and/or $\boldsymbol{c}$.

(a) $\overrightarrow{AC} =$ $\underset{\sim}{c} - \underset{\sim}{a}$ ✓

(b) $\overrightarrow{OB} =$ $\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c}$ ✓ $\overrightarrow{OA} + \overrightarrow{AB}$

(c) $\overrightarrow{BC} =$ $-\left[\frac{1}{2}\underset{\sim}{c} + \underset{\sim}{a}\right] + \underset{\sim}{c}$ ✓ $\overrightarrow{BO} + \overrightarrow{OC}$

$= \frac{1}{2}\underset{\sim}{c} - \underset{\sim}{a}$

(d) $\overrightarrow{MN} =$ $\overrightarrow{MC} + \overrightarrow{CN}$

$-\frac{2}{3}\underset{\sim}{a} + \underset{\sim}{c} - \frac{2}{3}\left(\frac{1}{2}\underset{\sim}{c} - \underset{\sim}{a}\right)$ ✓

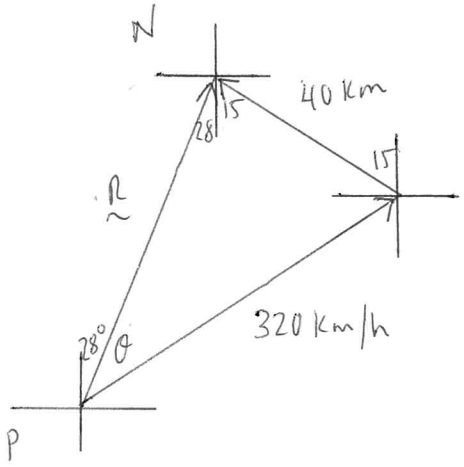
$= -\frac{2}{3}\underset{\sim}{a} + \underset{\sim}{c} - \frac{1}{3}\underset{\sim}{c} + \frac{2}{3}\underset{\sim}{a}$

$= \frac{2}{3}\underset{\sim}{c}$ ✓

6. (8 marks)

Frazier flies a light aircraft to Newman from Perth. The bearing of Newman from Perth is 028°T. His plane cruises at 320 km/h, but when he takes off, a wind blowing from 165°T at 40 km/h forces him to change his flight plan.

(a) Draw a clearly labelled vector diagram to show his flight path. [2]



(b) Calculate his **resultant** speed and the direction he should take to land directly in Newman. [5]

$$\frac{\sin\theta}{40} = \frac{\sin 43°}{320}$$ ✓

$\theta = 4.89°$ ✓ (28+5) ✓

∴ Bearing is 33.11°T [023°T]

$$\therefore |\underset{\sim}{R}| \Rightarrow \frac{|R|}{\sin 132.11°} = \frac{40}{\sin 4.89}$$ ✓

$|\underset{\sim}{R}| = 348.1$ km/h ✓

(c) If Newman is 980 km from Perth, determine the time Frazier's trip takes. [1]

$$\frac{980}{t} = 348.1$$

$t = 2.815$ hrs. ✓

11

7. (12 marks)

Consider the letters of the word EQUATION.

Given that repetitions are not permitted, how many arrangements of all the letters are possible if:

(a) there are no restrictions. [1]

$8! = 40320$

(b) the vowels are together. [2]

$4! \times 5!$

$= 2880$

(c) Q and U are not together. [2]

$8! - 7! \times 2!$

$= 30240$

2

(d) E is the first letter or N is the last letter. [3]

$7! + 7! - 6!$

$= 9360$

(e) E and N are separated by more than 1 letter. [4]

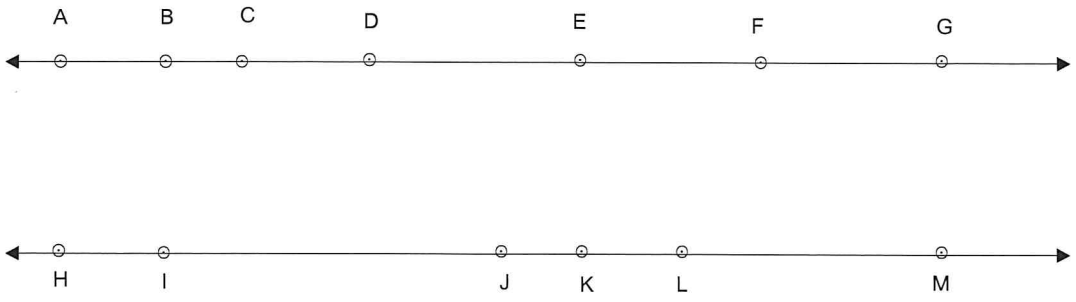
$8! - 7!(2) - 6! \times 6 \times 2$

$= 21600$

or $6! \times 5 \times 2 + 6! \times 4 \times 2 + 6! \times 3 \times 2 + 6! \times 2 \times 2 + 6! \times 2$

$= 21600$

8. (8 marks)



Triangles can be formed using the two parallel lines above, by choosing two points from one line and one point from the other line. Only the labelled points may be used.

(a) How many triangles in total are possible can be formed? [2]

$^7C_2 \times {}^6C_1 + {}^7C_1 \times {}^6C_2 = 231$ ✓✓

(b) How many triangles can be formed if two of the points forming the vertices of the triangle must come from the line containing A and G? [2]

$^7C_2 \times {}^6C_1 = 126$ ✓✓

(c) If the two points taken from the same straight line in forming the triangle must not be adjacent how many triangles can be formed? [3]

$[{}^7C_2 - 6] \times {}^6C_1 + [{}^6C_2 - 5] \times {}^7C_1$ ✓✓✓

$= 160.$

9. (4 marks)

Determine the number of three letter permutations of the letters of the word **SPECIALIST**

S P E C I A L T
S I

Case 1 → All different letters.

$8 \times 7 \times 6 = 336$ ✓

Case 2 → 2 same letter and one other.

$^2C_1 \times {}^7C_1 \times \frac{3!}{2!} = 42$ ✓✓

Total = 378 ✓